Simplified Models of the Angular Dependence of the Transmission of Woven or Perforated Screens

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Introduction

In our first approximation, woven shading layers are modeled as ideally diffusing planar reflectors* with a specular transmittance corresponding to the passage of radiation through the spaces between the weave. Simple geometric considerations indicate that this specular transmittance should have a dependence on the incident angle(s). Here we present those geometric considerations and define several simple models for estimating the angular dependence.

The geometry of a simple woven shade of evenly-spaced threads in a cross-sectional plane perpendicular to one of the thread directions is shown in Figure 1. (It is assumed that the threads of the weave run horizontally and vertically.) It can be seen from the figure that this simple geometry is characterized by the periodicity, **L**, of the weave, which here is the center-to-center distance between the threads, and the radius, **r**, of the thread. The projected transmittance at normal incidence in this two-dimensional model is simply the ratio

$$\frac{d_0}{L} = \frac{L \quad 2r}{L} = 1 \quad 2\frac{r}{L} \tag{1}$$

so that it is the ratio of these two quantities that defines the geometry. For radiation incident at an angle , it can easliy be seen from the figure that the projected opening between the threads is

$$d = L\cos \quad 2r \tag{2}$$

leading to a projected transmittance of

$$\frac{d}{L} = \begin{cases} \cos \quad \frac{2r}{L} & \text{for } \cos \quad \frac{2r}{L} \\ 0 & \text{for } \cos \quad < \frac{2r}{L} \end{cases} \tag{3}$$

This shows that in this projection there is a cutoff angle,

$$\cos_{cutoff} = \frac{2r}{L}. \tag{4}$$

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* We assume for simplicity that the threads are opaque, but nothing in this memo would prevent modeling a translucent screen as having a diffuse transmittance of radiation striking the material, in addition to the specular one for radiation passing through the holes.

It is convenient to rewrite these equations in terms of d_0 rather than r:

$$\frac{d}{L} = \begin{cases} \cos \quad 1 \quad \frac{d_0}{L}(& \text{for } \cos \quad) \quad 1 \quad \frac{d_0}{L}(\\ 0 & \text{for } \cos \quad < \quad 1 \quad \frac{d_0}{L}(\end{cases} \tag{5}$$

$$\cos \quad _{cutoff} = \quad 1 \quad \frac{d_0}{L}(\tag{6}$$

Zero-Parameter Models of Specular Transmittance

Square Weave

We can make models of the transmittance that require no additional parameters beyond the normal transmittance by assuming a regular square weave pattern as shown in Figure 2. The normal transmittance gives the necessary ratio,

$$T(0) = \quad \frac{d_0}{L} \quad ^2 \text{, or } \frac{d_0}{L} = \sqrt{T(0)}\,. \tag{7}$$

Applying equation 5 must take into account that the quantity d in the equation is a different function for the horizontal and vertical directions. If we subscript these with H and V, respectively, the equation for the transmittance for an incident direction $\mathbf{s}$ is

$$T(\mathbf{s}) = \quad \frac{d}{L} \quad _H \quad \frac{d}{L} \quad _V \tag{8}$$

The quantity $\mathbf{s}$ is a unit vector specifying the incident direction. The two d/L ratios in the equation are calculated from equation 5 using the projected angle, $_H$ or $_V$ in the horizontal or vertical plane, which is defined as the angle between the normal to the shade layer and the projection of $\mathbf{s}$ in the horizontal or vertical plane. For an incident direction with the angles $(\quad , \quad)$, where the azimuthal angle is defined with the $= 0$ axis in the horizontal plane,

$$\begin{aligned} \cos \quad _H &= \frac{\cos}{\sqrt{\sin^2 \quad \cos^2 \quad + \cos^2}} \\ \cos \quad _V &= \frac{\cos}{\sqrt{\sin^2 \quad \sin^2 \quad + \cos^2}} \end{aligned} \tag{9}$$

The transmittance as a function of incidence direction is then

$$T(\ ,\ ) = MAX\left\{\left(\cos_{H}\left(1\ \sqrt{T(0)}\right)\right)\left(\cos_{V}\left(1\ \sqrt{T(0)}\right)\right)\ ,\ 0\right\} \tag{10}$$

and the condition $T(\ ,\ ) = 0$ defines a cutoff contour in $(\ ,\ )$ that reflects the rectilinear geometry of the weave.

Perforated Screen

It is tempting to use equation 6 to define a simpler single-angle cutoff. This in effect assumes that the holes are cylindrically symmetric, i.e., circular. So this assumption actually corresponds to the physical situation shown in Figure 3, of a screen perforated with circular holes. Figure 3 shows the simplest such pattern (others could be produced by introducing additional parameters). But to be consistent, the transmittance at normal incidence should be defined differently for this geometry:

$$T(0) = \left(\frac{d_0}{L}\right)^2, \text{ or } \frac{d_0}{L} = \sqrt{\frac{T(0)}{\ }} \tag{11}$$

Then

$$T(\ ) = MAX\left\{\left(\cos_{\pm}\left(1\ \sqrt{\frac{T(0)}{\ }}\right)\right)^2\ ,\ 0\right\} \tag{12}$$

and

$$\cos_{cutoff} = \left(1\ \sqrt{\frac{T(0)}{\ }}\right) \tag{13}$$

A One-Parameter Model

The model can be extended to more general weave patterns by introducing additional parameters. The simplest case is that shown in Figure 4, where it is assumed that openings in the weave are separated by some (unknown) number of closely-spaced threads. Assuming that threads in a given direction do not cross one another, a measurement of the layer thickness, , (e.g., using a micrometer) will give the sum of the diameters of the horizontal and vertical threads, as indicated in the figure. For this situation the analog to equation 5 becomes

$$\frac{d}{L} = \begin{cases} \frac{d_0}{L}\cos + \frac{1}{2}\frac{(}{L}(\cos\ )\ 1) & \text{for } \cos \quad \dfrac{\frac{(}{L}}{2\frac{d_0}{L} + \frac{(}{L}} \\ 0 & \text{for } \cos < \dfrac{\frac{(}{L}}{2\frac{d_0}{L} + \frac{(}{L}} \end{cases}, \qquad (14)$$

from which it can be seen that the model depends on the new parameter $/L$. This cannot be deduced from the transmittance at normal incidence, since as can be seen this ratio drops out of equation 14 for $=0$. In addition to , one will also need to measure the periodicity length of the weave, L, e.g., using a calipers and magnifying glass, and to compute the ratio from these measurements.

Square Weave, Complex Weave Pattern

We can readily apply this model to a square weave pattern. Equation 7 still holds for the transmittance at normal incidence, and applying equation 14 to equation 8 gives

$$T(\ ,\ ) = MAX - \sqrt{T(0)}\cos_H + \frac{1}{2}\frac{}{L}\left(\cos_H\ 1\right) + \sqrt{T(0)}\cos_V + \frac{1}{2}\frac{}{L}\left(\cos_V\ 1\right),\ 0 \qquad (15)$$

where the projected angles are still calculated from equation 9.

Rectangular Weave

If the periodicity lengths for the two thread directions are not equal, then one has the situation illustrated in Figure 5. With no loss of generality we can take the horizontal periodicity length (i.e., the periodicity of the spacing of the vertical threads) to be L and the horizontal hole dimension d_0 as before. The corresponding vertical quantities are then expressed with two aspect ratios, x and y: $L_V = xL$ and $d_{0V} = yd_0$. Even for a simple weave pattern, x and y will in general be different. They can be determined by examining the weave using a calipers and magnifying glass.

Simple Weave Pattern

The transmittance at normal incidence is now

$$T(0) = \frac{y}{x}\left(\frac{d_0}{L}\right)^2 \text{, or } \frac{d_0}{L} = \sqrt{\frac{x}{y}T(0)} \tag{16}$$

and we can readily rewrite equation 10 as

$$T(\ ,\ ) = MAX\left(\pm\cos_H\left(1\sqrt{\frac{x}{y}T(0)}\right)\right)\left(\cos_V\left(1\sqrt{\frac{y}{x}T(0)}\right)\right),\ 0\left(-\right) \tag{17}$$

for this situation.

Complex Weave Pattern

A similar modification of equation 15 yields, for a rectangular weave with a complex weave pattern,

$$T(\ ,\ ) = MAX\left(-\sqrt{\frac{x}{y}T(0)}\cos_H + \frac{1}{2}\left(\frac{}{L}\right)\left(\cos_H\left(1\right)\right)\right) + \sqrt{\frac{y}{x}T(0)}\cos_V + \frac{1}{2x}\left(\frac{}{L}\right)\left(\cos_V\left(1\right)\right),\ 0 \tag{18}$$

Figures

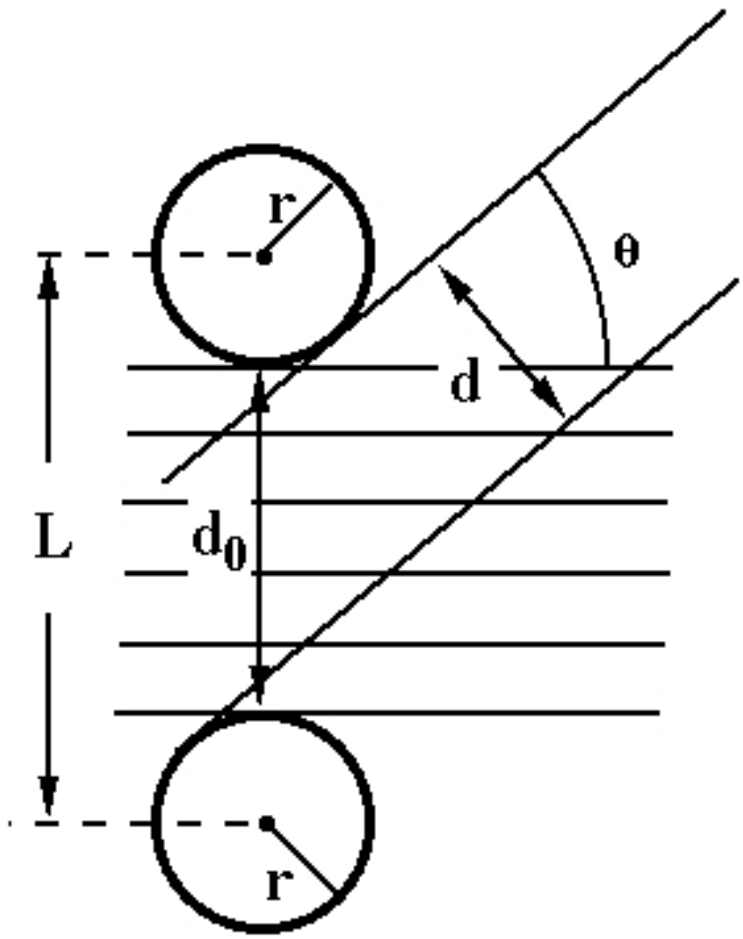

Figure 1. Simplified Two-Dimensional Geometry for a Simple Woven Screen. Horizontal lines indicate normal-incidence radiation.

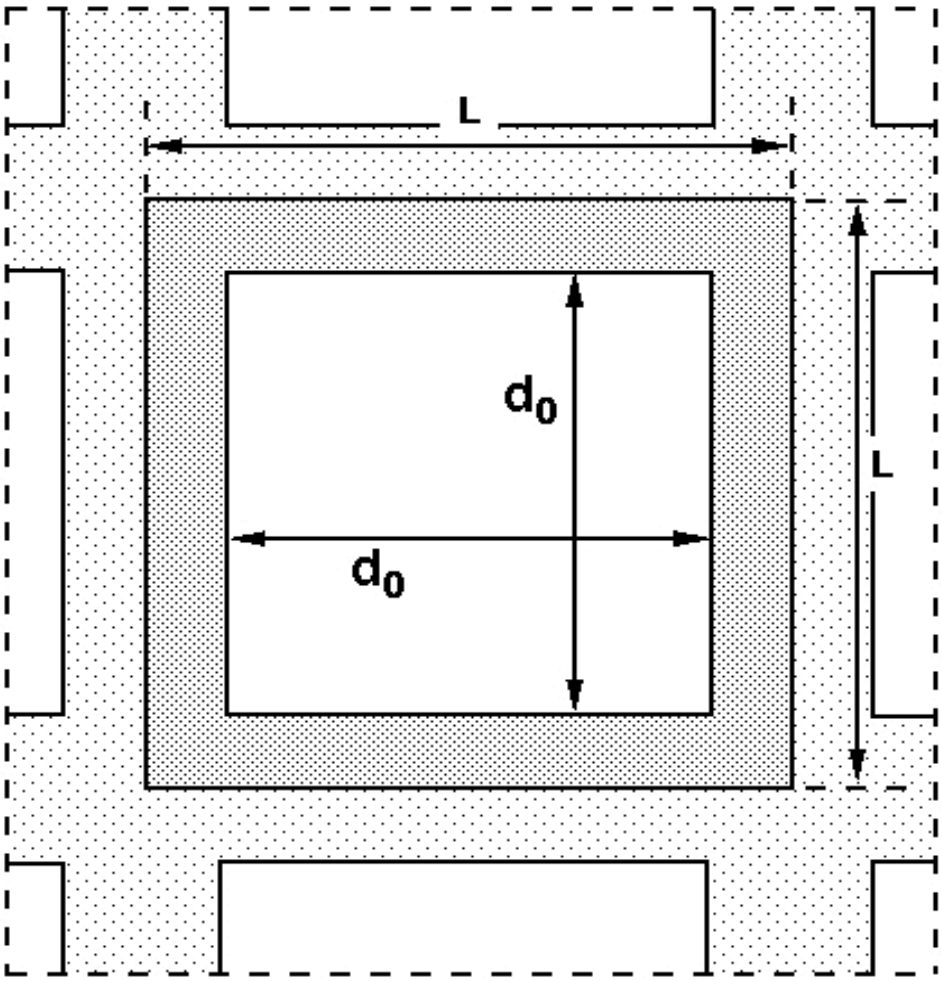

Figure 2. Typical Unit Cell of a Simple Square-weave Screen.

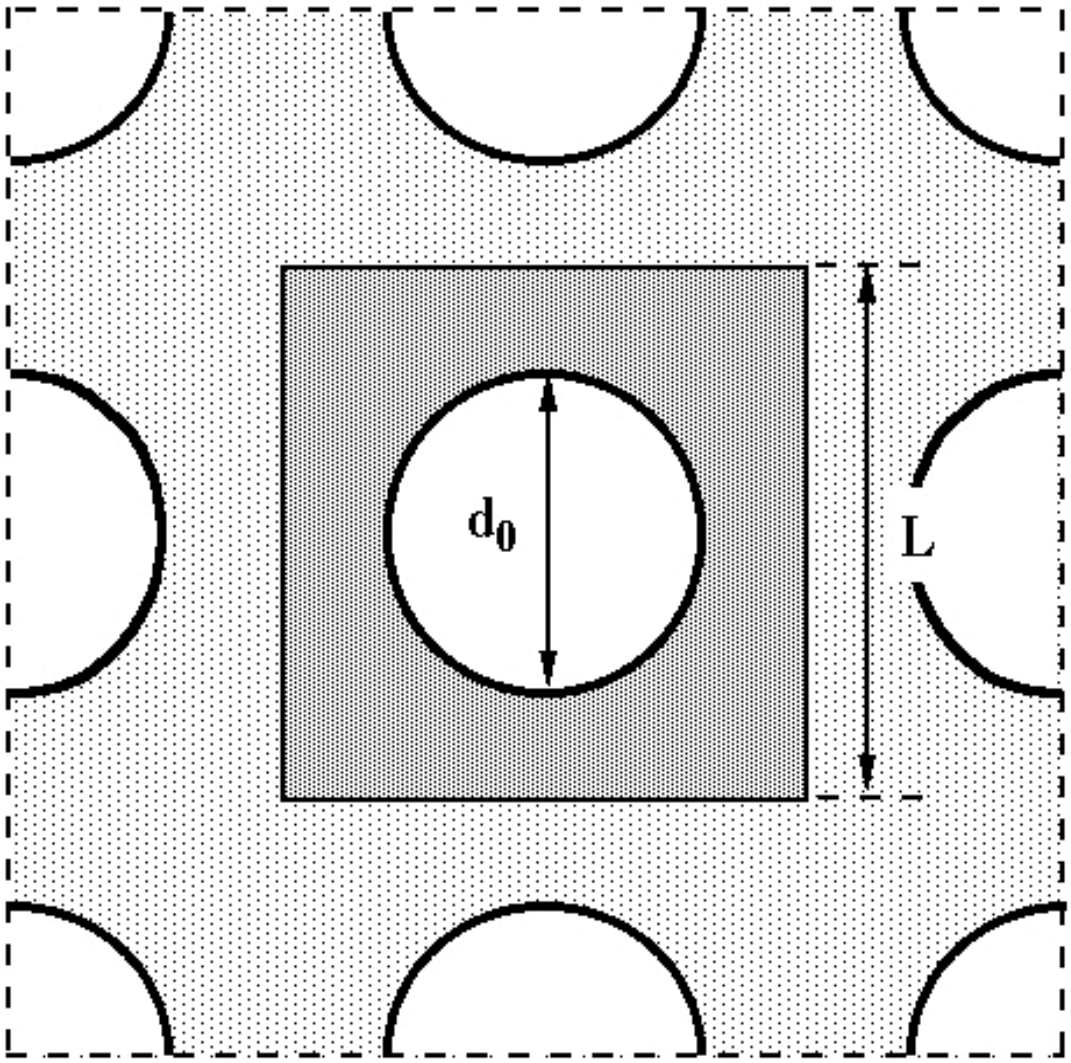

Figure 3. Unit Cell for an Opaque Screen with Circular Holes.

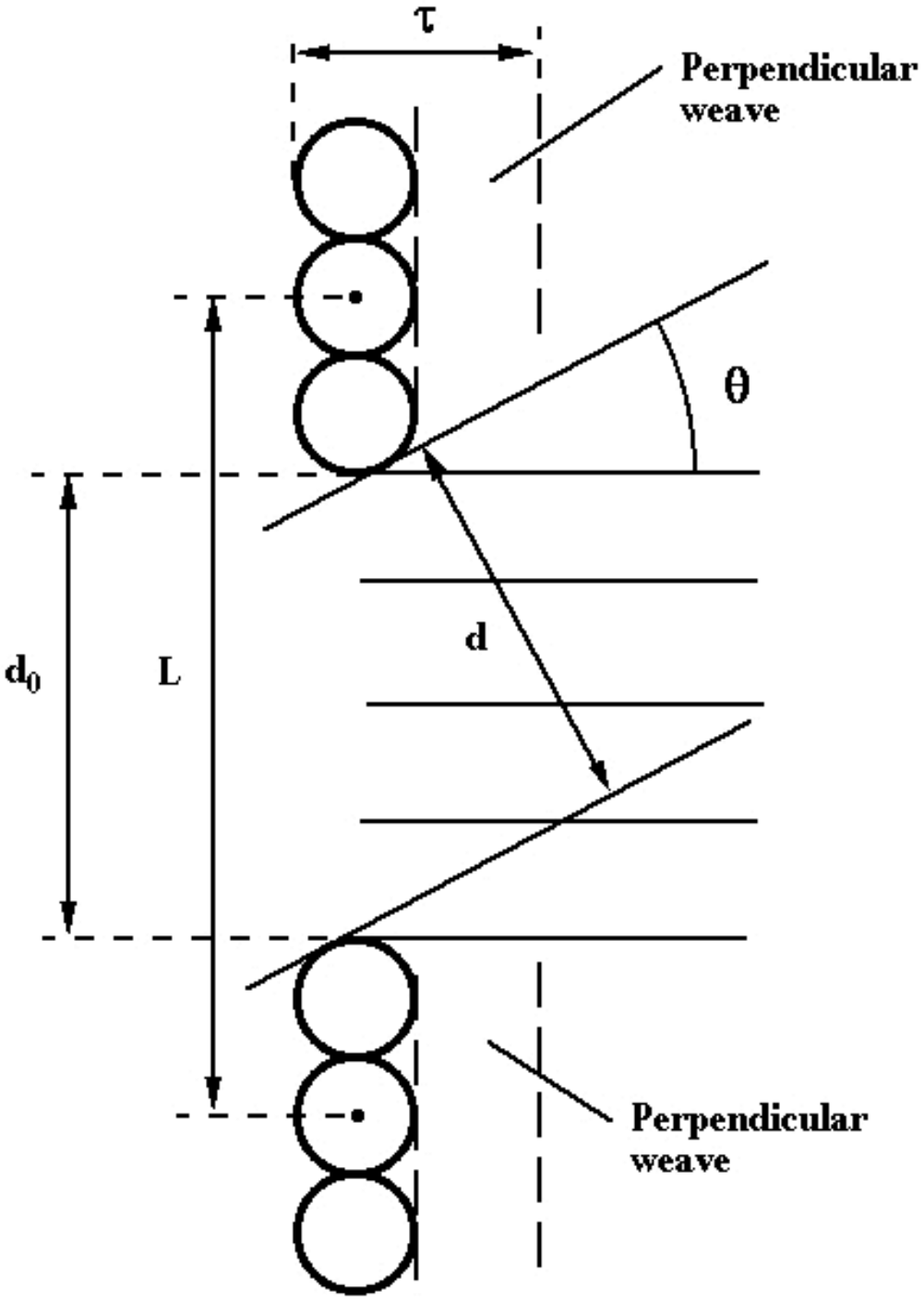

Figure 4. A Screen of Complex Weave. Openings alternate with multiple rows (here shown as three) of closely spaced threads.

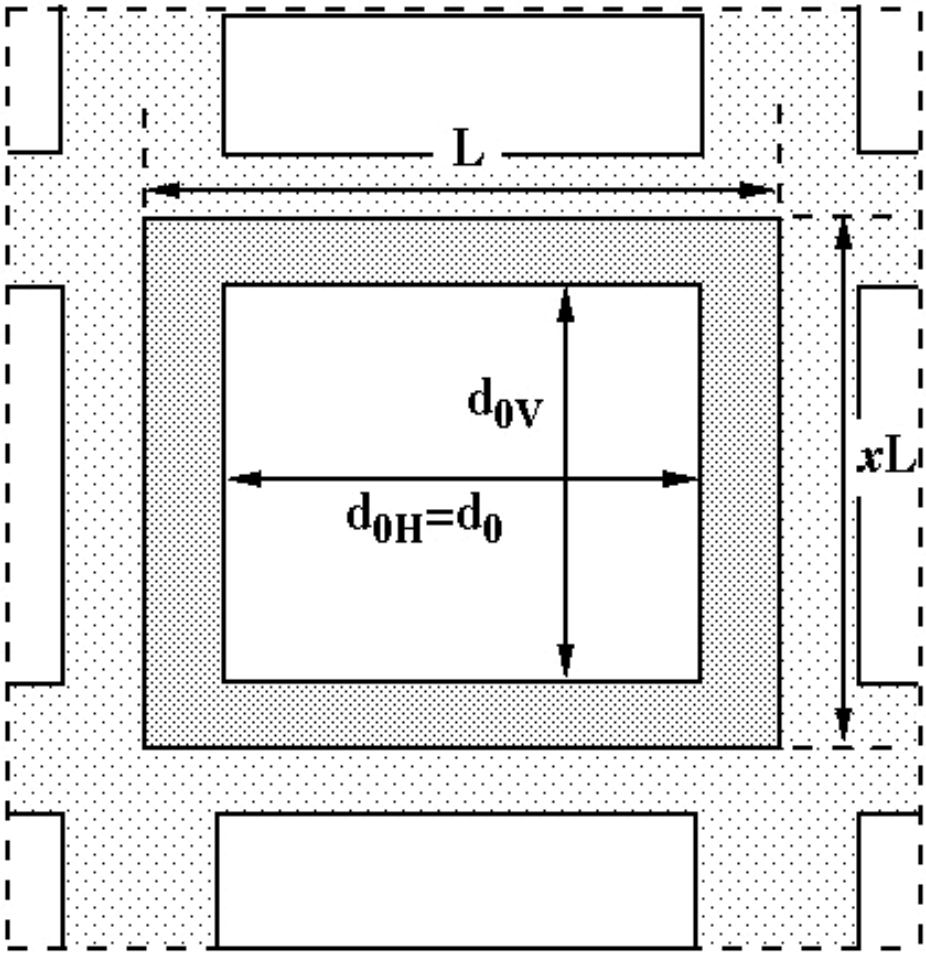

Figure 5. Unit Cell for a Rectangular Weave Pattern.