System Property Notation

capitol T means system transmittance

front

$$\mathbf{T}^{f}_{2\{i-1,i\}}$$

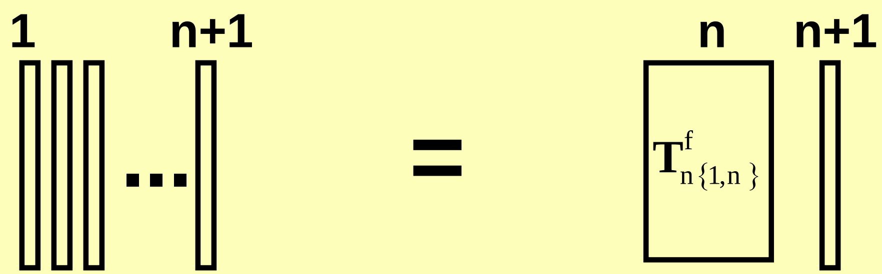
a 2-layer (sub) system

(sub)system includes layers i-1 thru i

Interreflections

- The sum-of-geometric series argument familiar in dealing with specular multiple reflections also works for the matrix calculation

Many-Layer Systems



subsystem back reflectance

$$\mathbf{T}^{f}_{n+1\{1,n+1\}} = \tau^{f}_{n+1} \cdot \left(\mathbf{1} - \Lambda \cdot \mathbf{R}^{b}_{n\{1,n\}} \cdot \Lambda \cdot \rho^{f}_{n+1}\right)^{-1} \cdot \Lambda \cdot \mathbf{T}^{f}_{n\{1,n\}}$$

a “recursion relation”

The Matrix Calculation Produces All of the Solar-Optical Quantities Necessary, Once the Layer Properties are Known

Conclusion

There is a consistent and systematic way to calculate the solar-optical properties of a fenestration system composed of an arbitrary number of layers, including interreflections, once the bidirectional properties of the layers are known

A New Method for Predicting the Solar Heat Gain of Complex Fenestration Systems

J. H. Klems

- ASHRAE/DOE Research Project
- Develop method for windows with non-specular, spatially nonhomogeneous element(s)
  - Shades, Drapes, Blinds
  - Figured glass
  - Transparent cellular glazings

Why is a New Method Needed?

- Increasingly sophisticated & accurate methods available for calculating performance of optically “simple” glazings
- No comparable methods available for optically complex fenestrations
  - data/method primarily for single glazing
  - only source of performance data is calorimeter measurement on entire system
  - no way to adapt measurements to different conditions
  - number of possible combinations of shading element, glazing types, coatings, tints growing rapidly
- Most windows have shading/privacy control

Key Technical Question

Can we find a method that

- **is accurate**
- **applies to any fenestration**
- **reduces the need for direct measurement**
  - **more calculation**
  - **easier/cheaper/faster measurement**
  - **calculation from measurement of more basic properties**

The Installment Plan

Part 1: Physical Basis of the Calculation Method

- Basic idea
- Project outline
- Technical Issue
- Approximations leading to a tractible calculation

Part 2: Matrix Calculation Method and Example

Part...

An Extended Solar Heat Gain Coefficient

- **Extend the original definition of SHGC**

$$F = \tau + N_I \alpha$$

- **to multilayer, direction-dependent systems:**

$$F(\theta,\phi) = T_{fH}(\theta,\phi) + \sum_{i=1}^{M} N_i A_{fi}(\theta,\phi)$$

system hemispherical transmittance

layer inward-flowing fraction

layer absorptance

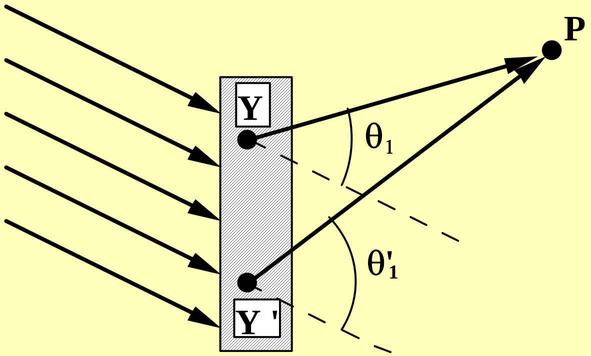
Two Types of Information Needed

- T_{fH}, A_{fi} are solar-optical properties
  - independent of temperature, longwave emissivity, etc.
- N_i are calorimetric quantities
  - depend on geometry, temperature, emissivity, etc.
  - independent of solar-optical properties (color, etc.)

Project Outline

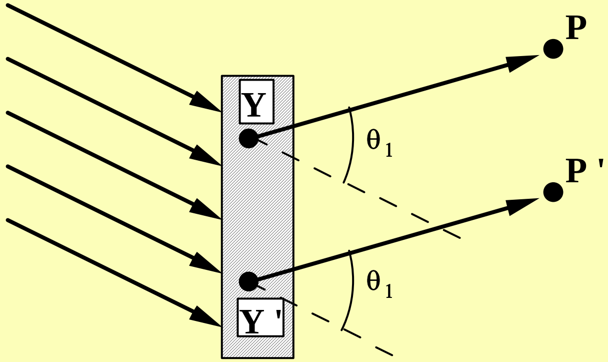
- **Develop method of calculating system solar-optical properties T_{fH}, A_{fi} from layer properties**
- **Develop method of measuring bidirectional solar-optical properties of nonspecular, spatially nonhomogeneous layers**
- **Measure inward-flowing fractions N_i for thermally prototypic geometries**
- **Calculate system SHGC from T_{fH}, A_{fi}, N_i**
- **Compare calculated SHGC with calorimeter measurement**

Optically Complex Layers



- **Radiation arrives at P**
  - **from many points in the layer**
  - **at many different angles**

Approximation 1: Fixed-angle Spatial Averaging

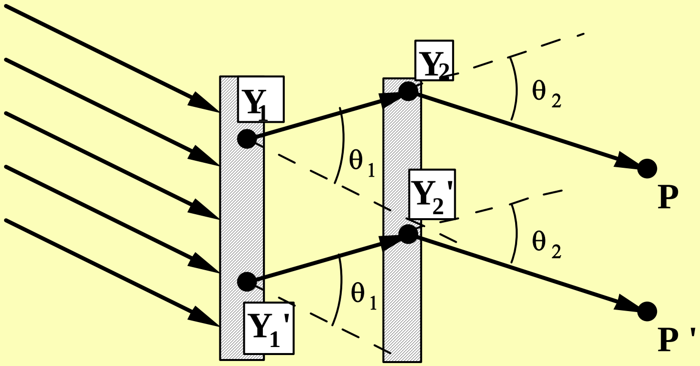


- **Hold** θ φιξεδ

∞ Σπατιαλ αϖεραγε οϖερ ποιντσ Ψ, Ψ↔

- **averaging region large compared to spatial inhomogeneity (quasi-periodic)**
- **destroys image information**
- **preserves total energy flow**

Approximation 2: Neglect Spatial Correlations Between Layers



$$I(P) = \int\int d\theta_2 d\theta_1 T_2(Y_2, \theta_2; \theta_1) T_1(Y_1, \theta_1; \theta_0) I(\theta_0)$$

- **Spatial averaging done separately for each layer**
  - **different layers cannot have commensurate scale of spatial structure (e.g., not two horizontal blinds)**

Approximation 3: Spectrally Averaged Properties

- Project concentrates on angular dependence
  - large-sample radiogoniometry
- Convenient starting assumption (simplicity)
  - better than current state of the art
  - may be reasonable for many systems
  - clearly fails if too many selective layers
- Not a necessary assumption
  - could extend to spectroradiogoniometry later

Resulting Simplification (2-layer system)

outgoing radiance

solid angle

area projection

layer transmittances

incident irradiance

$$I(\theta_2, \phi_2) = \int d\Omega_1 \cos(\theta_1)\tau_2^f(\theta_2, \phi_2; \theta_1, \phi_1)\tau_1^f(\theta_1, \phi_1; \theta_0, \phi_0)E(\theta_0, \phi_0)$$

- **reduced from 5 ™ 2 degrees of freedom/layer**
- **next:**
  - **need a way to carry out the calculation**
  - **must include interreflections between layers (omitted above)**

To be continued...

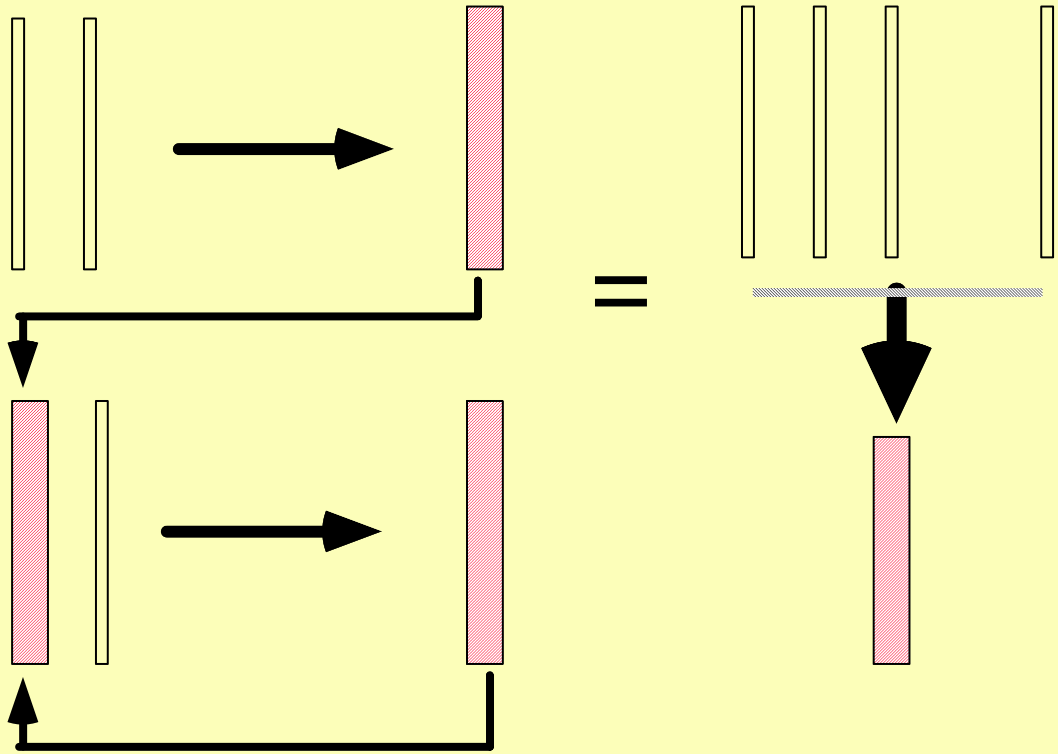
Layer Reflectance Matrix

$$\rho_i^f = \begin{Bmatrix} \rho_i^f(\theta_i^{r,1}, \phi_i^{r,1}; \theta_{i-1}^1, \phi_{i-1}^1) & \rho_i^f(\theta_i^{r,1}, \phi_i^{r,1}; \theta_{i-1}^2, \phi_{i-1}^2) & \dots & \rho_i^f(\theta_i^{r,1}, \phi_i^{r,1}; \theta_{i-1}^N, \phi_{i-1}^N) \\ \rho_i^f(\theta_i^{r,2}, \phi_i^{r,2}; \theta_{i-1}^1, \phi_{i-1}^1) & \rho_i^f(\theta_i^{r,2}, \phi_i^{r,2}; \theta_{i-1}^2, \phi_{i-1}^2) & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \rho_i^f(\theta_i^{r,N}, \phi_i^{r,N}; \theta_{i-1}^1, \phi_{i-1}^1) & \dots & \dots & \rho_i^f(\theta_i^{r,N}, \phi_i^{r,N}; \theta_{i-1}^N, \phi_{i-1}^N) \end{Bmatrix}$$

Backward-going reflected radiance:

$$\mathbf{J}_i = \rho_i^f \cdot \mathbf{E}_{i-1}$$

Recursive Calculation of Multilayer System Properties



Elements of the Transmittance Matrix are Biconical Transmittance Distribution Functions

$$\tau_{12} = \frac{\text{Outgoing Radiance } (\theta_1, \phi_1)}{\text{Incoming Irradiance } (\theta_2, \phi_2)}$$

Matrix Formulation of Nonspecular Transmittance

Outgoing Radiance Distribution

Incident Irradiance Distribution

$$\begin{matrix} I(\theta^1, \phi^1) \\ I(\theta^2, \phi^2) \\ \ldots \\ I(\theta^N, \phi^N) \end{matrix} = \left.\begin{matrix} \tau_{11} & \tau_{12} & \ldots & \tau_{1N} \\ \tau_{21} & \tau_{22} & \ldots & \ldots \\ \ldots & \ldots & \text{etc}\ldots & \ldots \\ \tau_{N1} & \ldots & \ldots & \tau_{NN} \end{matrix}\right\} \cdot \begin{Bmatrix} E(\theta^1, \phi^1) \\ E(\theta^2, \phi^2) \\ \ldots \\ E(\theta^N, \phi^N) \end{Bmatrix}$$

Transmittance Matrix