Carli, Inc.
18 Tanglewood Rd.
Amherst, MA 01002
TEL: (413) 256-4647
FAX: (413) 256-4823
home@fenestration.com
http://www.fenestration.com

DRAFT PROPOSAL:

IMPLEMENTATION OF WOVENSHADE METHOD IN LAYEROPTICS.DLL

Project: **WinCOG**

Module: **LayerOptics.dll**

DLL versions: *6.1.24*

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1. INTRODUCTION

The purpose of this document is to present a proposal for implementation of a calculation of bi-directional solar optical properties of woven shade devices in LayerOptics.dll, which is based on the direct transmittance model by Klems (2006) and diffuse reflectance model by FSEC (McCluney 2006). This methodology will be implemented in a function called WovenShade. Input arguments for this function are geometry of a woven shade device, diffuse reflectance of the shade material, and definition of directions of interest, for which bi-directional properties of the device will be calculated. Output arguments are four matrices – BTDF (Bi-directional Transmittance Distribution function) and BRDF (Bi-directional Reflectance Distribution function), with bi-directional properties at the front and back side of the device.

2. DEFINITION OF ANGLE COORDINATES

Figure **1** shows two coordinate systems: *xyz* (we will refer to it as a "forward" system) and $x^r y^r z^r$ ("reversed" or "reflected" system).

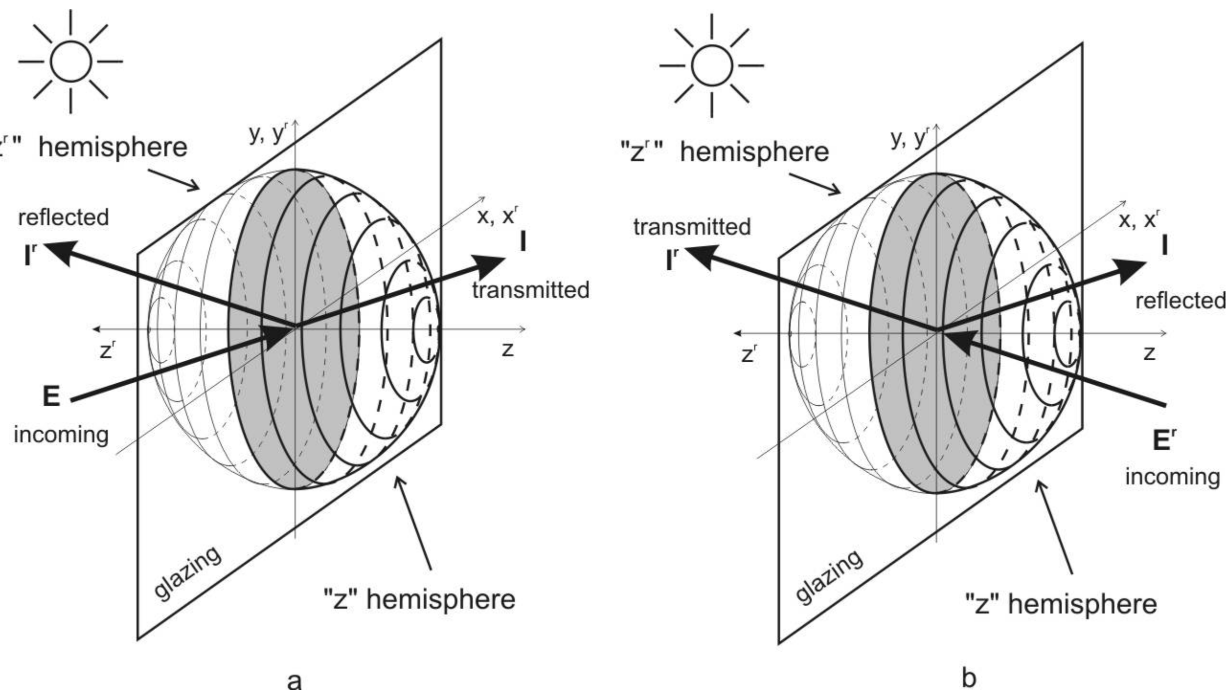

Figure 1. Two hemispheres, with corresponding coordinate systems

Axis *z* points towards the indoor side, while axis z^r points towards the outdoor side. These two axes define 2 hemispheres. All directions that correspond to radiation that travels forward ("left-to-right" with regards to the woven shade, where 'left' is outdoor side and 'right' is indoor side) are defined in *xyz* coordinate system, using "*z*" hemisphere (incident and transmitted radiation – *E* and *I* in Figure **1**a, or reflected radiation *I* in Figure **1**b), and directions that correspond to backward-going radiation ("right-to-left" or toward the outdoor side) are defined in the reflected $x^r y^r z^r$ coordinate system, using "z^r" hemisphere (reflected radiation I^r in Figure **1**a, or incident and transmitted radiation E^r and I^r in Figure **1**b).

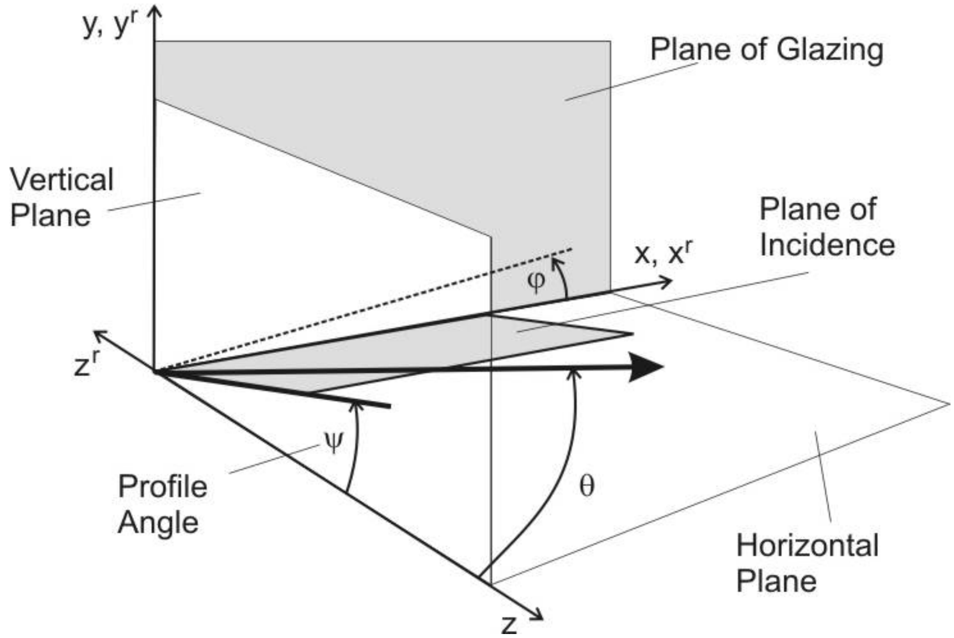

Figure 2. Angles definitions – forward-going light

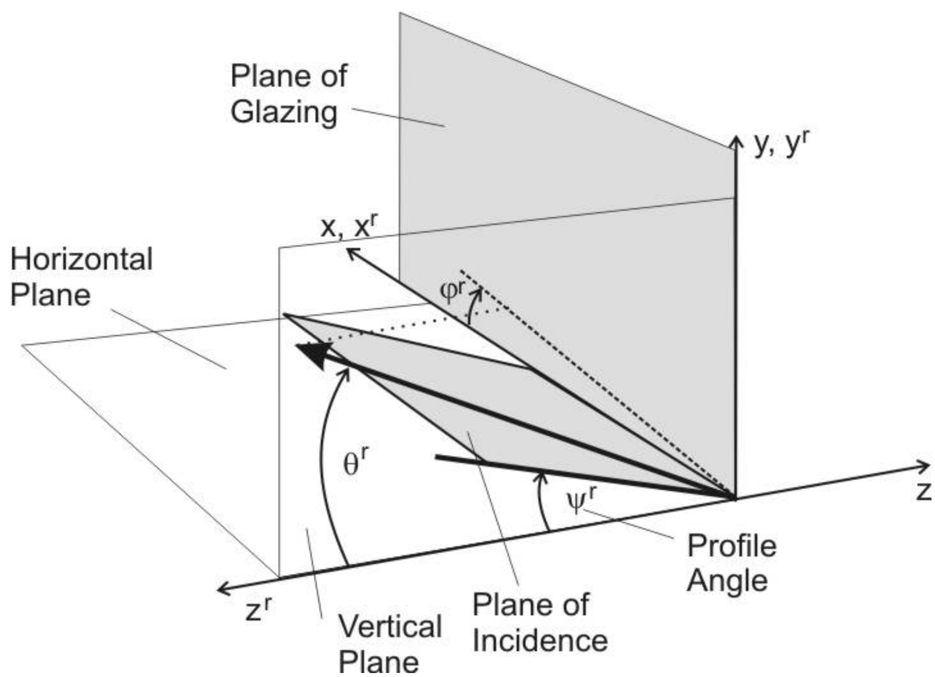

Figure 3. Angles definitions – backward-going light

Directions of interest are defined using angular coordinates θ (latitude angle) and φ (azimuth angle) for forward-going radiation (Figure **2**), and θ^r and φ^r for backward going radiation (Figure **3**). Latitude angle θ is the angle between z axis and the direction of interest, φ is the angle between *x* axis and *x0y* projection of the direction of interest. Angle θ^r is the angle between z^r axis and the direction of interest, and angle φ^r is in fact equal to φ, because *x* and *y* axes are the same in both coordinate systems. To illustrate this: $\theta = 0°$ lies on the *z* axis; all directions defined as ($\theta > 0°$, $\varphi = 0°$) lie in the horizontal (*x0z*) plane; all directions defined as ($\theta > 0°$, $\varphi = 90°$) lie in the vertical (*y0z*) plane etc. Values of θ and φ are defined within following limits:

$$\begin{aligned} 0° &\le \theta < 90° \\ 0° &\le \varphi < 360° \end{aligned} \tag{1}$$

Same restrictions apply to θ^r and φ^r.

Figure **4** shows an example with directions of interest in planar projection of the "*z*" hemisphere in *x0y* plane, with *z* axis pointing towards the viewer. The diameters of the circles representing θ angles are growing with θ value. Value of angle φ grows in positive (counter-clockwise) direction. Numbers 1 – 49, shown in Figure 4, correspond to a set of pre-defined bins, defined by 7 θ angles and 8 φ angles. This set of 49 bins had been given only as an example to illustrate the concept, since full angular set consists of 145 bins. In this example, direction 15 (or D_{15}) is defined as ($\theta_{15} = 45°$, $\varphi_{15} = 225°$). For backward directions, Figure **4** can also be used: axes x^r and y^r are oriented the same as axes *x* and *y*, and axis z^r points away from the viewer. In this frame of reference, numbers in Figure **4** correspond to backward-going directions.

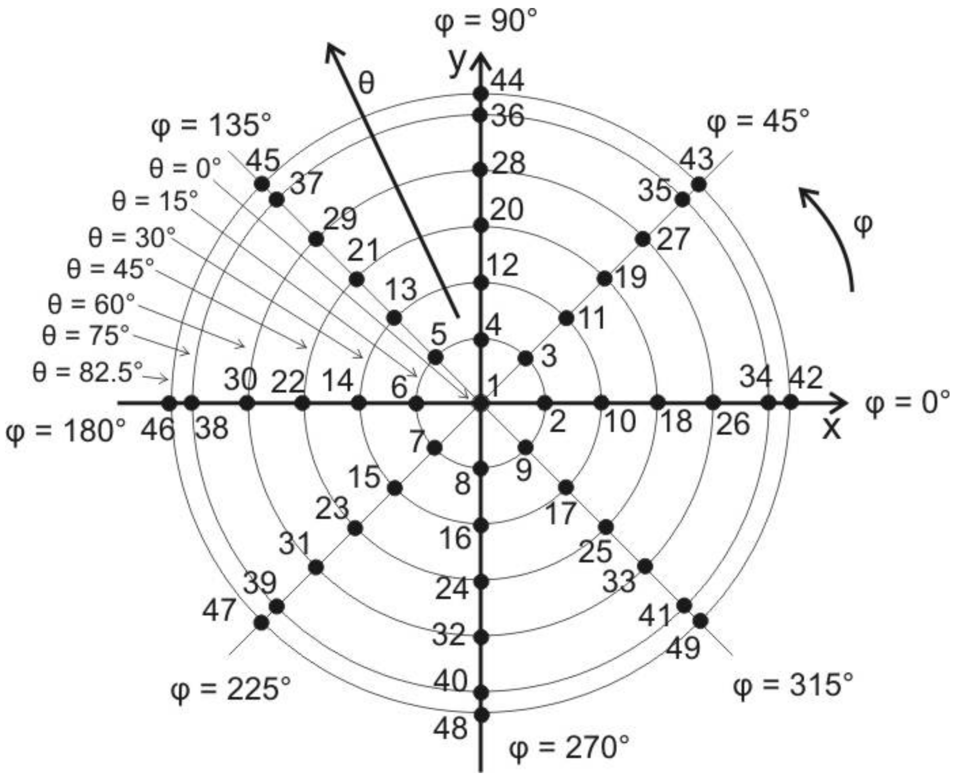

Figure **4.** Projection of "*z*" hemisphere in *x0y* plane, viewed from the indoor side

Each direction *i*, in forward or reversed coordinates, is determined by its angular coordinates, as (θ_i ,φ_i) or (θ^r_i ,φ^r_i), where i = 1, … , N_{dir} (number of directions).

The calculations for woven shades are essentially done in 2-D space, ignoring edge effects in x and y directions. Transition from 3D to 2D space is carried out through introduction of profile angles ψ (for forward-going radiation) and ψ^r (for backward-going radiation). Profile angle is the angle between the plane of incidence and horizontal plane, when projected on a vertical plane. Profile angles for forward- and backward-going directions are shown in Figure **2** and Figure **3**, respectively. The connection between angular coordinates and profile angle in general is given by:

$$\tan\psi = \sin\varphi \cdot \tan\theta \tag{2}$$

One of the consequences of using angular coordinates defined in two coordinate systems as shown in Figure **2** and Figure **3** is that incident directions are being defined in the "z" hemisphere. For example, incident direction coming from the outdoor side from above the horizontal plane will actually be defined in the "z" hemisphere. Therefore, associated profile angle for this direction calculated using equation (2) will have a negative value. For incident radiation, profile angles will be defined as:

$$\begin{aligned} \psi_i &= -\arctan\left(\sin\varphi_i \cdot \tan\theta_i\right) \\ \psi_i^r &= -\arctan\left(\sin\varphi_i^r \cdot \tan\theta_i^r\right) \end{aligned} \tag{3}$$

where ψ_i is profile angle of the i-th incident direction for forward-going radiation, and ψ_i^r is profile angle of the i-th incident direction, for backward-going radiation.

For outgoing angles, profile angles will be defined as:

$$\begin{aligned} \psi_j &= \arctan\left(\sin\varphi_j \cdot \tan\theta_j\right) \\ \psi_j^r &= \arctan\left(\sin\varphi_j^r \cdot \tan\theta_j^r\right) \end{aligned} \tag{4}$$

where ψ_j is profile angle of the j-th outgoing direction for forward-going radiation, and ψ_j^r is profile angle of the j-th outgoing direction, for backward-going radiation.

All 2D calculations are performed using these profile angles, and results are stored in resulting bi-directional matrices (BTDF/BRDF matrices) in places that correspond to the particular (θ, φ) bin and its associated profile angle.

3. DIRECT TRANSMITTANCE MODEL

The geometry of a simple woven shade of evenly-spaced threads in a cross-sectional plane perpendicular to one of the thread directions is shown in Figure 1. (It is assumed that the threads of the weave run horizontally and vertically.) It can be seen from the figure that this simple geometry is characterized by the periodicity, L, of the weave, which here is the center-to-center distance between the threads, and the radius, r, of the thread. The projected transmittance at normal incidence in this two-dimensional model is simply the ratio d0.

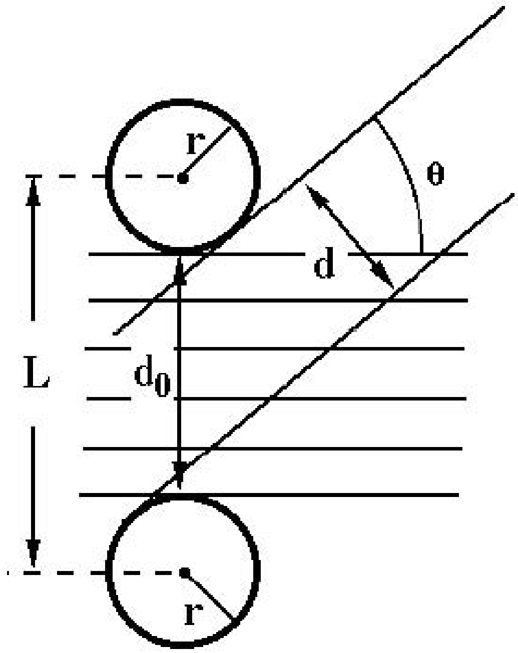

Figure 5. Simplified Two-Dimensional Geometry for a Simple Woven Screen. Horizontal lines indicate normal-incidence radiation – Taken from Klems (2006)

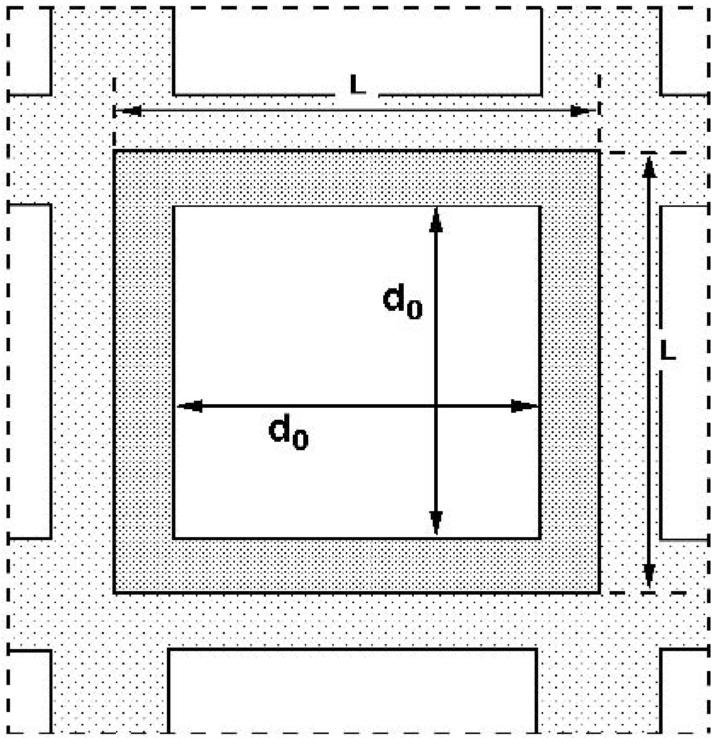

Figure 6. Typical Unit Cell of a Simple Square-weave Screen – Taken from Klems (2006)

By assuming a regular square weave pattern as shown in Figure 2. The normal transmittance gives the necessary ratio,

$$T(0) = \left(\frac{d_0}{L}\right)^2$$

For an incident direction with the angles (θ,φ), where the azimuthal angle is defined with the φ= 0 axis in the horizontal plane,

$$\cos\theta_H = \frac{\cos\theta}{\sqrt{\sin^2\theta\cos^2\varphi + \cos^2\theta}}$$

$$\cos\theta_V = \frac{\cos\theta}{\sqrt{\sin^2\theta\sin^2\varphi + \cos^2\theta}}$$

And the transmittance as a function of incidence direction is then:

$$T(\theta,\varphi) = MAX\left\{\left[\cos\theta_H - \left(1-\sqrt{T(0)}\right)\right]\cdot\left[\cos\theta_V - \left(1-\sqrt{T(0)}\right)\right], 0\right\}$$

And the condition T(θ,φ) = 0 defines a cutoff contour in (θ,φ) that reflects the rectilinear geometry of the weave.

Using the same procedure to calculate directly transmitted radiation, McCluney (2006) gives comparison between the result of such analytical model and ray tracing, as shown in Figure 7.

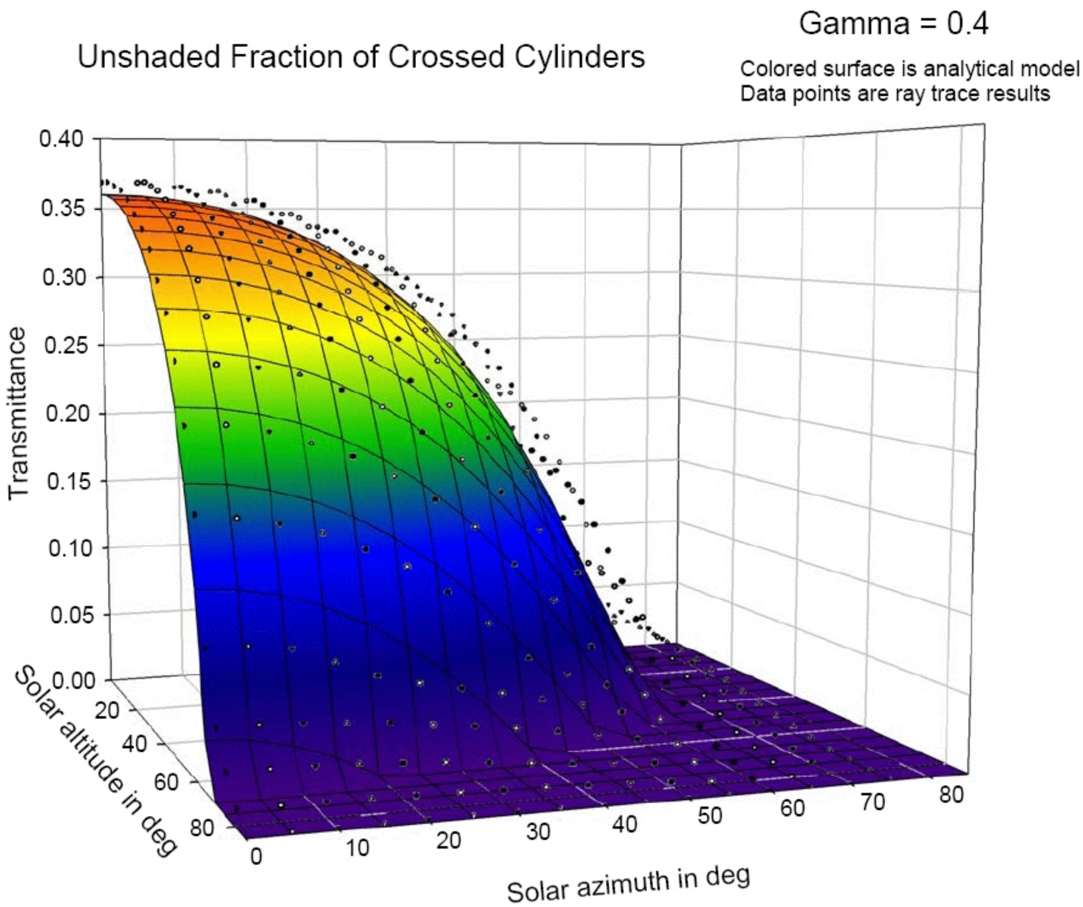

Figure 7. Analytical model output for Unshaded Fraction (transmittance) – Taken from McCluney (2006)

4. DIFFUSE REFLECTANCE MODEL

FSEC window screen model [McCluney, R. , DOE 2006 (1)] calculates diffuse (scattered) transmittance using geometry of woven shade material and relative position of the sun with respect to the surface outward$T_{scatter}$, a part of beam solar radiation which is reflected inward from cylinder's surface.

The scattering contribution to transmittance in Figure 8 is equivalent to $T_{scatter}$. Figure 8 shows the additive component for scattered transmittance for a screen material having an aspect ratio of 0.4 and a material reflectance of 0.4. Although this component is shown to be highly directional, the magnitude (0.055) is small compared to the transmittance of a totally absorbing surface. For this reason, the scattered transmittance due to a reflective screen material surface will be added directly to the transmittance of a totally-absorbing screen material (or could be considered as beam-to-diffuse transmittance). Figure 8 shows distribution of $T_{scatter}$ for Aspect ratio 0.4.

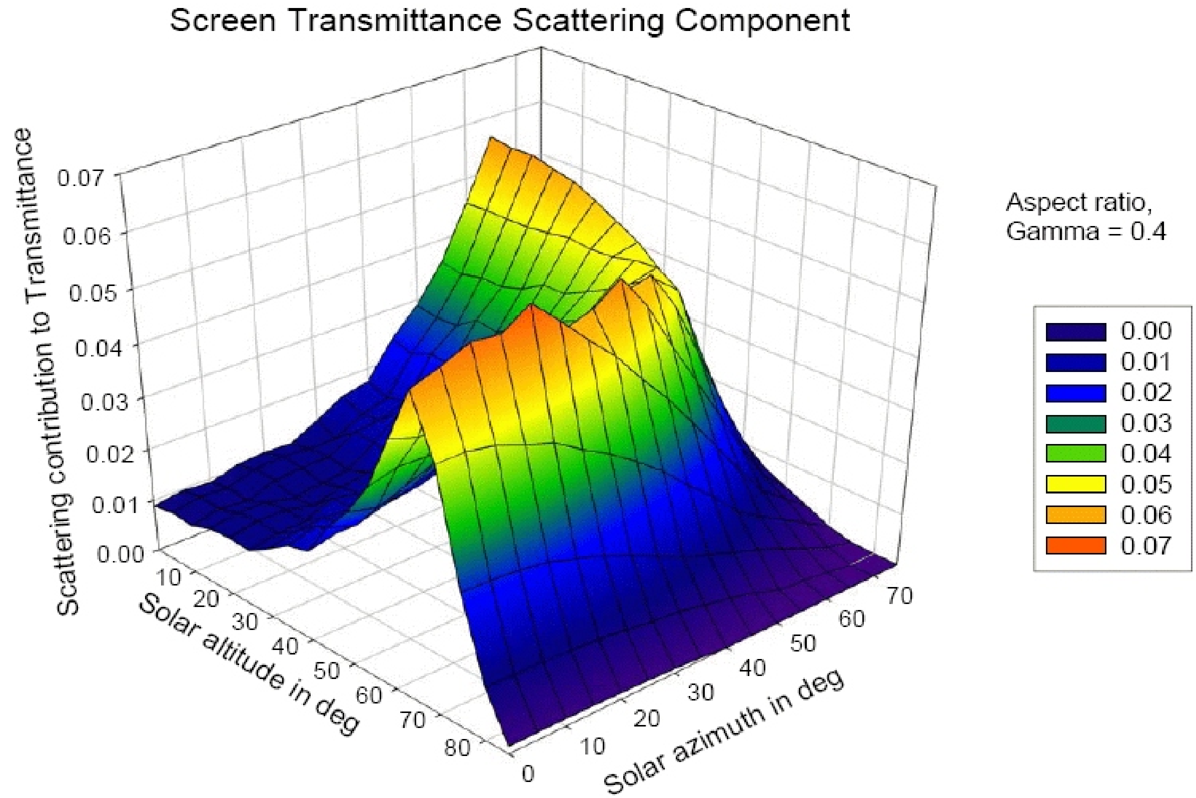

Figure 8. Transmittance due to reflective property of screen material

Diffuse transmittance is calculated each time step based on sun position and surface orientation. To account for the variation in front surface reflectance for varying sun angles, a simplified model is used. The front surface reflectance model is based on the geometry of the screen material and the reflectance property of the screen material cylinders (**Error! Reference source not found.**). The following equations are used for the calculation of T_{scatter}:

$$T_{scatter} = 0.6(1-\gamma)(T_{\text{scattermax}})\left(1+3e^{\frac{-|\delta-\delta_{\max}|^{1.7}}{600}}\right)$$

$$\delta = \sqrt{1.2\omega^2 + 0.7\sigma^2}$$

$$\delta_{\max} = 89.7 - 10\cdot\left(\frac{\gamma}{0.3}\right)$$

$$T_{scatter\,\max} = 0.015\cdot r\cdot\left(1-\left(\frac{\gamma-3}{6}\right)\right)$$

If ($\delta > \delta_{\max}$); $T_{\text{scatter}} = T_{\text{scatter}} - 0.2\cdot$`$\gamma\cdot r\cdot T_{\text{scattermax}}$

$$\gamma = D/S$$

$$\omega = MAX\left(0, MIN\left(\frac{\pi}{2}, Solar\ Altitude + \left(\frac{\pi}{2} - Surface\,Tilt\right)\right)\right)$$, Surface Tilt > 0

$$\sigma = MAX\left(0, MIN\left(\frac{\pi}{2}, ABS(Solar\ Azimuth - Surface\ Azimuth)\right)\right)$$, Surface Tilt > 0

Where the following relative angles of the sun with respect to the surface outward normal are defined:

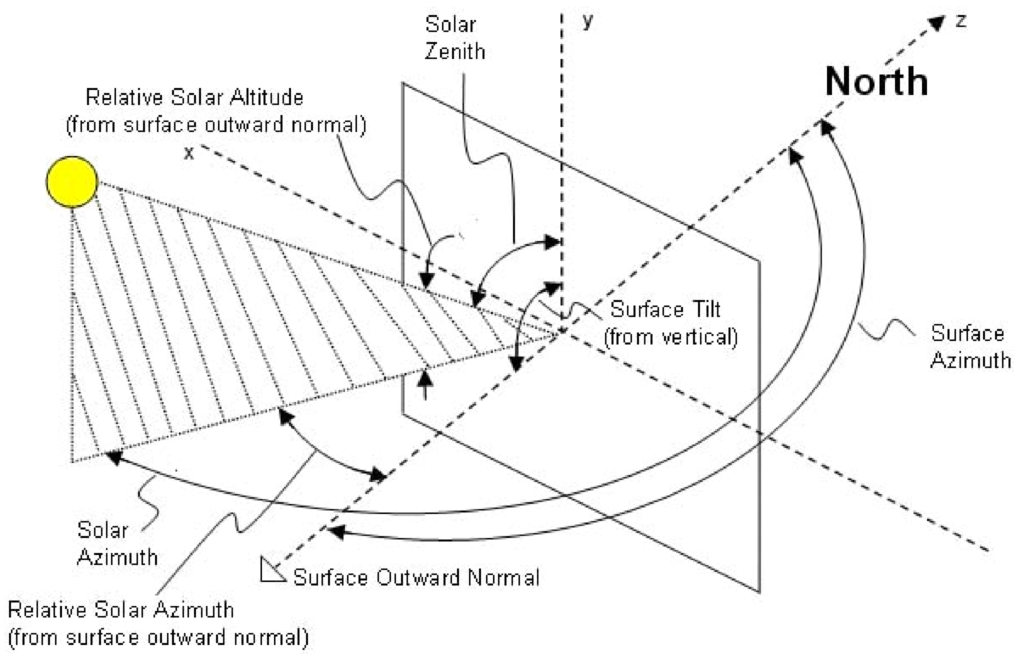

Figure 9.Schematic of window screen facing due south

FSEC model calculates reflectance of the screen as:

$$R_{scatter} = r \cdot (1 - T_{direct}) - T_{scatter} \quad (1)$$

where *r* is reflectance of window screen material.

It should be noted that the form of this equation in McCluney (2006) is somewhat different due to an error in the derivation. Within the simplifying assumptions, this is the correct version of the equation.

The overall absorptance of the woven shade material is then:

$$A = 1 - (T_{direct} + T_{scatter} + R_{scatter})$$

Which reduces to:

$$A = (1 - r) \cdot (1 - T_{direct})$$

5. WOWENSHADE FUNCTION

WovenShade function will calculate bi-directional properties - transmittance and reflectance, which are defined for each combination of incident and outgoing direction, and as a result will form BTDF and BRDF matrices.

WovenShade function will perform calculations according to Klems and FSEC model. Since FSEC model provides properties for a single incident direction, WovenShade method must perform a series of calculations in order to produce bi-directional properties for all incident and outgoing directions. Directions of interest will be defined as in Venetian function.

For a given incident direction *i*, defined by the pair of angle coordinates ($Theta_i$, Phi_i), transmittances $T_{direct,i}$ and $T_{scatter,i}$ are calculated. In order to get bi-directional transmittance, $T_{direct,i}$ must be divided with Lambda factor before putting into appropriate place in the result matrix, which is at the diagonal (as directly transmitted component exists only for the outgoing direction that is collinear with the incident direction). Lambda is propagation operator which transforms radiance vector emerging from one layer into irradiance vectors incident on the next layer. $T_{scatter,i}$ which is a hemispherical property, must be divided with Pi in order to obtain bi-directional transmittance (Pi transforms diffuse outgoing radiance into direct outgoing radiance). These values will be placed in *i*-th column of a BTDF matrix, which corresponds to *i*-th incident direction.

This procedure is repeated for each incident direction. Direct and scattered components of bi-directional transmittances are summed to form **TAU_F** matrix, (equation 2).

$$\mathrm{TAU\_F} = \begin{bmatrix} \frac{T^f_{direct,1}}{\Lambda_1} + \frac{T^f_{scatter,1}}{\pi} & \frac{T^f_{scatter,2}}{\pi} & \cdots & \frac{T^f_{scatter,N}}{\pi} \\ \frac{T^f_{scatter,1}}{\pi} & \frac{T^f_{direct,2}}{\Lambda_2} + \frac{T^f_{scatter,2}}{\pi} & \cdots & \frac{T^f_{scatter,N}}{\pi} \\ & & \cdots & \\ \cdots & \cdots & \cdots & \cdots \\ \frac{T^f_{scater,1}}{\pi} & \frac{T^f_{scatter,2}}{\pi} & \cdots & \frac{T^f_{direct,N}}{\Lambda_N} + \frac{T^f_{scatter,N}}{\pi} \end{bmatrix} \qquad (2)$$

The same procedure is applied for backward incident directions. **TAU_B** matrix, (equation 3) consists of backward bi-directional transmittances.

$$\mathrm{TAU\_B} = \begin{bmatrix} \frac{T^b_{direct,1}}{\Lambda_1} + \frac{T^b_{scatter,1}}{\pi} & \frac{T^b_{scatter,2}}{\pi} & \cdots & \frac{T^b_{scatter,N}}{\pi} \\ \frac{T^b_{scatter,1}}{\pi} & \frac{T^b_{direct,2}}{\Lambda_2} + \frac{T^b_{scatter,2}}{\pi} & \cdots & \frac{T^b_{scatter,N}}{\pi} \\ & & \cdots & \\ \cdots & \cdots & \cdots & \cdots \\ \frac{T^b_{scatter,1}}{\pi} & \frac{T^b_{scatter,2}}{\pi} & \cdots & \frac{T^b_{direct,N}}{\Lambda_N} + \frac{T^b_{scatter,N}}{\pi} \end{bmatrix} \qquad (3)$$

Scattered reflectances $R_{scatter,i}$ are hemispherical values, calculated according to equation (1). As in the case of scattered transmittance, these values must be divided with Pi in order to get bi-directional reflectances. The **RHO_F** matrix (equation 4) contains front bi-directional reflectances, while **RHO_B** matrix (equation 5) contains back bi-directional reflectances.

$$\mathrm{RHO\_F} = \begin{bmatrix} \frac{R^f_{scatter,1}}{\pi} & \frac{R^f_{scatter,2}}{\pi} & \dots & \frac{R^f_{scatter,N}}{\pi} \\ \frac{R^f_{scatter,1}}{\pi} & \frac{R^f_{scatter,2}}{\pi} & \dots & \frac{R^f_{scatter,N}}{\pi} \\ & & \dots & \\ \dots & \dots & \dots & \dots \\ \frac{R^f_{scatter,1}}{\pi} & \frac{R^f_{scatter,2}}{\pi} & \dots & \frac{R^f_{scatter,N}}{\pi} \end{bmatrix} \quad (4)$$

$$\mathrm{RHO\_}B = \begin{bmatrix} \frac{R^b_{scatter,1}}{\pi} & \frac{R^b_{scatter,2}}{\pi} & \dots & \frac{R^b_{scatter,N}}{\pi} \\ \frac{R^b_{scatter,1}}{\pi} & \frac{R^b_{scatter,2}}{\pi} & \dots & \frac{R^b_{scatter,N}}{\pi} \\ & & \dots & \\ \dots & \dots & \dots & \dots \\ \frac{R^b_{scatter,1}}{\pi} & \frac{R^b_{scatter,2}}{\pi} & \dots & \frac{R^b_{scatter,N}}{\pi} \end{bmatrix} \quad (5)$$

5.1. Input and Output Arguments

Input arguments of WovenShade function

- *D* - cylinder diameter [m]
- *S* - cylinder spacing [m]
- *r* - diffuse reflectance of the screen material
- *Theta* - relative solar altitude angle [degrees]
- *Phi* - relative solar azimuth angle [degrees]
- *NumThetas* - Number of incident angles Theta
- *NumPhis* - Array with number of Phi angles for each Theta {NumThetas}

Outputs are four square matrices: **TAU_F** , **TAU_B** , **RHO_F** and **RHO_B** (as defined in previous section) with *N* columns and rows, where *N* is the number of directions of interest. Element *i, j* of these matrices corresponds to appropriate bi-directional property (transmittance or reflectance), in which incident beam comes from *j*-th direction and scatters in *i*-th direction.

6. REFERENCES

DOE 2006 (1), EnergyPlus Engineering Reference

DOE 2006 (2), EnergyPlus Input/Output Reference

Carli, Inc. 2006, Calculation of Optical Properties for a Venetian Blind Type of Shading Device, Technical Report

McCluney, R. 2006, "New Feature Proposal: Material: WindowScreen." . Florida Solar Energy Center. January 31, 2006.