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Optical Model of Fritted Glazing in WINDOW

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1. INTRODUCTION

Fritted glazing (also known as “frits”) is a special type of glazing which consists of a glazing substrate (usually specular glass) and diffuse patterns on the glass, commonly referred to as frits. The frit layer consists of glass and in some cases colored pigments that are melted to bond to the substrate surface. This process can easily be masked so that a pattern of the fritted surface is created. Because of the presence of diffuse material, the whole glazing layer is considered to be a complex glazing, which requires a more sophisticated calculation of the glazing system optical properties , which incorporates fritted glazing layer. These more complex calculations also require more detailed optical data, because the incoming direction of solar radiation is no longer equal to the outgoing direction, which is the case in specular glazing layers. The optical data for each complex glazing layer is provided in four matrices, one for front transmittance, one for back transmittance, one for front reflectance and one for back reflectance. Each matrix has data for 145 incident directions and 145 outgoing directions for each incident direction, therefore forming 145 x 145 matrix of data, or 21,025 data points. These matrices are provided for each wavelength.

In the case of a highly spectrally selective layer, which is characterized by 300 wavelengths, the total amount of data is 4 x 21,025 x 300 or 215,230,000 data points. In comparison, specular glazing layer of the same spectral selectivity will have 300 transmittances, 300 front reflectances and 300 back reflectances, for a total of 900 data points. Note that for a specular glazing layer, only one set of transmittance data is needed, as front and back transmittances in specular transmission are identical.

The WINDOW software, developed by Lawrence Berkeley National Laboratory, implements calculations for fritted glazing using the matrix data discussed above. In WINDOW, spectral data for fritted glazing is stored in the WINDOW database in a table called *SpectralData*. In order to generate spectral data for a fritted glazing layer, measured optical data from the frit substrate glazing layer without the frit and from the frit substrate with the frit covering are combined using special algorithms, called “Fritted glazing composition”, detailed in this report. Fritted glazing composition optical calculations use a two layer optical model. Data for each layer is stored in a database. First layer is for substrate data, second layer is for frits. However, frits can have diffuse and specular portion. Both diffuse and specular portion of frits are obtained by importing data from two different files (one for front side and one for back side measurements), for a total of four individual optical data files.

Columns of interest stored in *SpectralData* table are:

- *Wavelength*
- *T*
- *Rf*
- *Rb*
- *Tb*

This document describes algorithms and procedures for fritted glazing composition optical calculations.

2. OPTICAL DATA

When loading spectral data for frits, the following arrays are used to store the data in the program:

- *FritSpectralDataTf*
- *FritSpectralDataTb*
- *FritSpectralDataRf*
- *FritSpectralDataRb*
- *FritSpectralWL*

The introduction of transmittance back is necessary since unlike for specular glazings, the transmittance does not have to be identical from the front and the back. All these matrices are three dimensional with the following dimensions:

1. (Glazing) Layer
2. Component
3. Index

(Glazing) Layer is used to denote fritted glazing layer position in an IGU. For example, if fritted glazing layer is glazing layer 2 in an IGU, then (Glazing) Layer index will be equal to 1 (note that numbering starts from zero).

Component part is used to store layers data representing Frit layer. Since there are three layers used to represent one Frit, there will be three components stored in these arrays:

Component = 0 – Substrate data

Component = 1 – Specular data

Component = 2 – Diffuse data

Index is used to represent data for each wavelength

2.1 BSDF Data Model

Each fritted glazing layer has its optical data stored in Bi-Directional Scattering Function, or BSDF format. BSDF are matrices of transmittance and reflectance, where each matrix contains columns of incident directions and rows of outgoing directions. At the maximum, BSDF is 145 x 145 matrix, corresponding to the 145 incident and 145 outgoing directions (solid angles). For smaller angular basis, the number of directions is reduced. Currently we have half basis (72 x 72 matrix) and quarter basis (36 x 36 matrix). Half and quarter basis may be used in calculations for improved speed, but the optical data for a layer is still stored in full 145 x 145 matrix. One BSDF Matrix is created for each wavelength. The number of wavelengths and BSDF directions is specified in WINDOW->Preferences->Optical Calcs screen (see Figure 1).

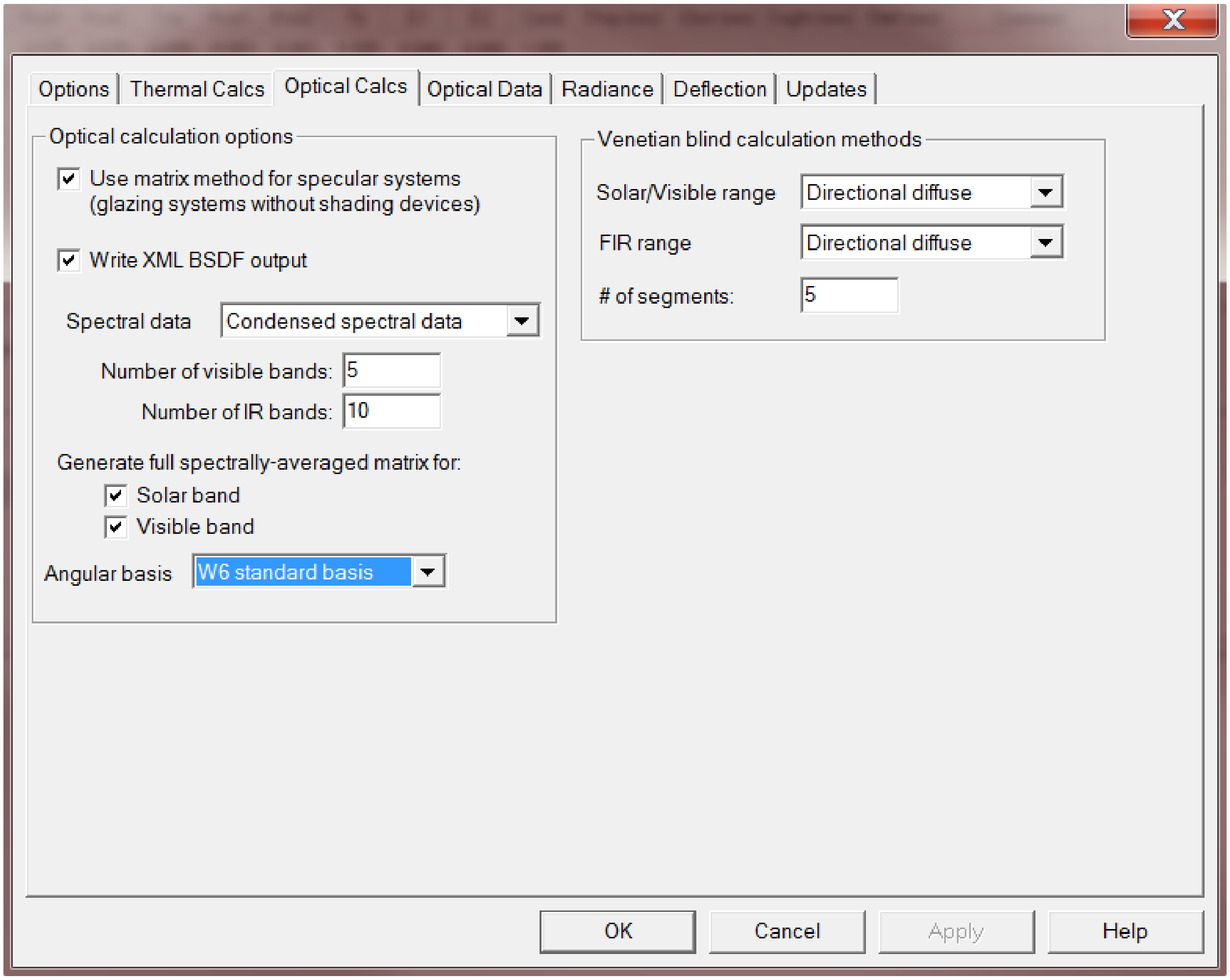

Figure 1: Options in "Optical Calcs" Tab Under "Preferences" Screen

Optical calculations in WINDOW, using Matrix calculation approach (mandatory for scattering glazing layers or shading devices) can be done for different wavelength ranges and for different angular bases definitions, as specified in Optical Calcs Preferences. Since frits are considered non-specular (i.e., diffuse) glazing layers, the matrix calculations will be performed independent of the setting for "Use matrix method for specular systems (glazing systems without shading devices)".

2.2 Specular Properties

Matrices for specular layer will have zero values, except for the values on the diagonal. The diagonal values will be calculated by using following equation:

$$BTDF_{(i)} = \frac{T(\lambda,i)}{\Lambda_i} \qquad \text{Eq.1}$$

Where $BTDF_{(i)}$ represents the BSDF value for transmittance in the i-th direction, Λ_i represents the solid angle partition for i-th direction and $T(\lambda, i)$ is transmittance for a specific wavelength (read from the spectral data tables) and adjusted for specific angular dependence. Note that Eq.1~~Eq.1~~ is valid for back and front transmittance as well as for front and back reflectance. Matrix presentation of Eq.1~~Eq.1~~ is:

$$BTDF = \left\| \begin{matrix} \frac{T(\lambda,1)}{\Lambda_1} & 0 & \dots & 0 \\ 0 & \frac{T(\lambda,2)}{\Lambda_2} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \frac{T(\lambda,N)}{\Lambda_N} \end{matrix} \right\| \qquad \text{Eq.2}$$

2.3 *Fritted Glazing Composition Optical Calculations*

Fritted glazing composition optical calculations consist of the following steps:

1. Specular optical properties of the frit layer readout
2. Diffuse optical properties of the frit layer readout
3. Create resultant frit layer matrix from specular and diffuse optical properties
4. Substrate optical properties readout
5. Create fritted glazing BSDF by area weighting

After performing above steps transmittance and reflectance (front and back) BSDF matrices (i.e., $BTDF_F$, $BTDF_B$, $BRDF_F$, $BRDF_B$) are created and are ready to be incorporated into the calculation of the glazing system, that may involve one or more specular glazing layers.

2.3.1 Specular Optical Properties of the Frit Layer

Specular optical properties are stored in a diagonal matrix, as described in section 2.2. Note that all four matrices are stored using the same diagonal matrix form.

2.3.2 Diffuse Optical Properties of the Frit Layer

For diffuse portion of the BSDF there is no angular dependence. This means that transmittance and reflectance is uniform and identical for all angles and is only wavelength -dependent. The values on the diagonal of this matrix are not used in the creation of the resultant matrix, and can be ignored.

Note: Because specular optical data is measured by subtracting diffuse from total radiation, where diffuse radiation is measured by opening port in an integrating sphere, the specular component intrinsically includes diffuse portion in the specular direction (See Section 2.6).

$$BTDF_{res} = \left\| \begin{matrix} 0 & \frac{T(\lambda)}{\pi-\Lambda_2} & \cdots & \frac{T(\lambda)}{\pi-\Lambda_N} \\ \frac{T(\lambda)}{\pi-\Lambda_1} & 0 & \cdots & \frac{T(\lambda)}{\pi-\Lambda_N} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{T(\lambda)}{\pi-\Lambda_1} & \frac{T(\lambda)}{\pi-\Lambda_2} & \cdots & 0 \end{matrix} \right\| \qquad \text{Eq.3}$$

Eq.3~~Eq.3~~ is valid for front and back transmittance as well as for front and back reflectance.

2.3.3 Frit Layer Optical Matrix

The resultant matrices of optical properties for the frit layer are created by adding specular and diffuse matrices. The resultant matrix contains terms in a diagonal which are equal to specular values, while rest of matrix is filled with the diffuse part.

$$BTDF = \left\| \begin{matrix} \frac{T(\lambda,1)}{\Lambda_1} & \frac{T(\lambda)}{\pi-\Lambda_2} & \cdots & \frac{T(\lambda)}{\pi-\Lambda_N} \\ \frac{T(\lambda)}{\pi-\Lambda_1} & \frac{T(\lambda,2)}{\Lambda_2} & \cdots & \frac{T(\lambda)}{\pi-\Lambda_N} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{T(\lambda)}{\pi-\Lambda_1} & \frac{T(\lambda)}{\pi-\Lambda_2} & \cdots & \frac{T(\lambda,N)}{\Lambda_N} \end{matrix} \right\| \qquad \text{Eq.4}$$

And again, this matrix is also valid for reflectance.

2.3.4 Substrate Optical Properties

Fritted glazing substrate data have the same format as the specular portion of the frit layer (Section 2.2). Only diagonal terms are non-zero.

2.3.5 Final Fritted Glazing Optical Properties

Substrate matrix is added to frit layer optical matrix (see Section 2.3.3) by weighting area of the frit coverage. If frit layer matrix is marked as $BTDF(\lambda)_{res}$ and substrate matrix is marked as $BTDF(\lambda)_{sub}$ then final fritted glazing BSDF is created using following equation:

$$BTDF(\lambda)_{final} = FritCoverage * BTDF(\lambda)_{res} + (1 - FritCoverage) * BTDF(\lambda)_s \qquad \text{Eq. 5}$$

Where *FritCoverage* is number between 0 and 1 and denotes area fraction of frit coverage (i.e., 0.2 is 20% frit coverage, 0.5 is 50% frit coverage, etc.).

2.4 Angular Dependence

Each term in the diagonal of the specular matrices correspond to an optical property at a given incidence angle. Since specular properties are only dependent on the overall incidence angle, without the consideration of solar azimuth, the only angular dependence is given with respect to incidence angle θ. Angular dependence for specular glazing is not measured, but is rather calculated. Calculation details are given in Reference [1] and will not be repeated here.

2.5 Hemispherical Transmittance

Hemispherical transmittance is an integrated property at the outgoing direction, representing overall optical properties for each incidence angle (i.e., each fritted glazing will have 145 hemispherical data points for each optical property (i.e., 145 Transmittances, 145 Reflectances). The hemispherical transmittance is calculated for each incident direction by using the following equation:

$$T_{hem,i} = \frac{\sum_{j=1}^{N} \int T_{i,j} * cos(\theta_j) * d\omega_j}{\sum_{j=1}^{N} \int cos(\theta_j) * d\omega_j} \quad \text{Eq.6}$$

Where $T_{i,j}$ is transmittance between incoming direction "i" and outgoing direction "j", θ_j is angle between surface normal and direction "j", N is number of outgoing directions and both integrations in Eq.6~~Eq.6~~ are performed over outgoing direction.

2.5.1 Solid Angle Transformation

In order to calculate integration given by Eq.6~~Eq.6~~, spherical coordinate system is used. In order to do integration in spherical coordinate system, infinitesimal angle dω needs to be transferred into spherical coordinate system. Since all outgoing rays are covered by outgoing half-sphere, integration of solid angle over half-sphere should be equal to 1. Therefore, we can calculate sold angle by simple division of dA and A:

$$d\omega = \frac{dA}{A} \quad \text{Eq.7}$$

Where A is area of half-sphere while dA is area of infinitesimal surface covered by solid angle dω.

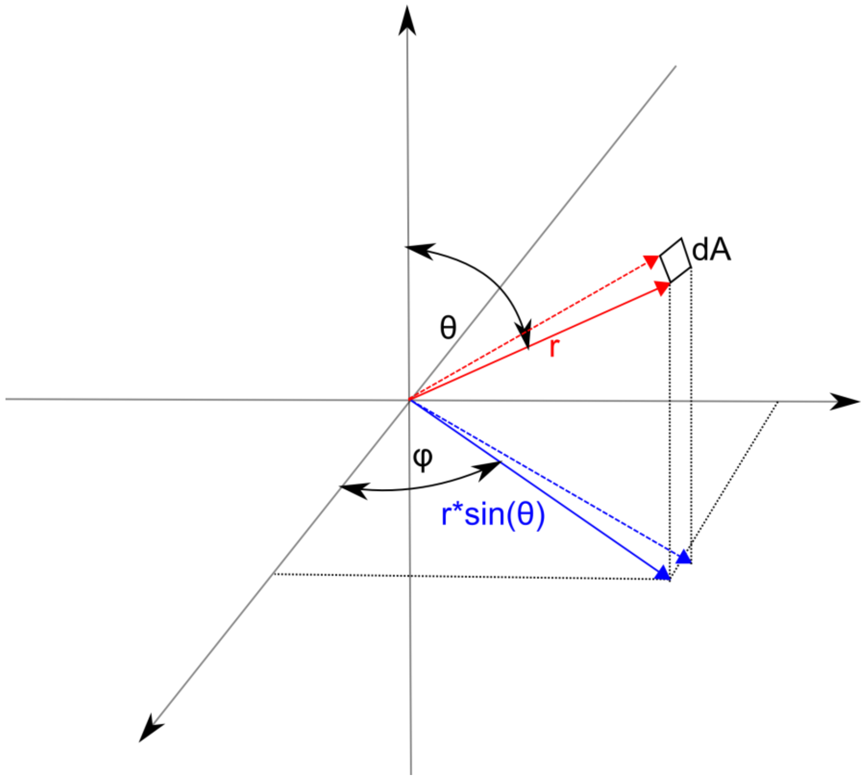

Figure 2: Spherical Coordinates

Area of surface *dA* is calculated by following equation:

$$dA = d\theta * r * \sin(\theta) * d\varphi * r \qquad \text{Eq.8}$$

and since half-sphere surface is equal to $2\pi r^2$, Eq.7~~Eq.7~~ becomes:

$$d\omega = \frac{1}{2\pi}\sin(\theta)\, d\theta d\varphi \qquad \text{Eq.9}$$

2.5.2 Hemispherical Transmittance Integration

Integration of Eq.6~~Eq.6~~ can be performed in two parts: nominator and denominator.

Integration over denominator is fairly simple and it can be represented with the following equation:

$$\sum_{j=1}^{N} \int cos(\theta_j) * d\omega_j = \int cos(\theta) * d\omega = \frac{1}{2\pi}\int_{\theta=0}^{\pi/2}\int_{\varphi=0}^{2\pi} cos(\theta)\sin(\theta)\, d\theta d\varphi \qquad \text{Eq.10}$$

Substituting back into Eq.6~~Eq.6~~ and integrating over direction “j”:

$$T_{hem,i} = 2 * \sum_{j=1}^{N} \int T_{j,i} * cos(\theta_j) * d\omega_j = 2 * \sum_{j=1}^{N} \int_{\theta=\theta_{j,LO}}^{\theta=\theta_{j,HI}} \int_{\varphi=\varphi_{j,LO}}^{\varphi=\varphi_{j,HI}} T_{j,i} * cos(\theta_j) * \qquad \text{Eq.11}$$

then replacing $d\omega_j$ with Eq.9~~Eq.9~~ and performing both integrations, the following equation is obtained:

$$T_{hem,i} = \frac{1}{2}\sum_{j=1}^{N} \tau_{j,i}\left((\sin\theta_{j,HI})^2 - (\sin\theta_{j,LO})^2\right)\Delta\varphi_j$$ Eq.12

where $\tau_{j,i} = \frac{T_{j,i}}{\pi}$ and $\Delta\varphi_j = \varphi_{j,HI} - \varphi_{j,LO}$.

Introducing lambda coefficient as:

$$\Lambda_j = \frac{1}{2}\left((\sin\theta_{j,HI})^2 - (\sin\theta_{j,LO})^2\right)\Delta\varphi_j$$ Eq.13

then Eq.12Eq.12 can be written as:

$$T_{hem,i} = \sum_{j=1}^{N} \tau_{j,i}\Lambda_j$$ Eq.14

It is also noteworthy that following equation is valid:

$$\pi = \sum_{j=1}^{N} \Lambda_j$$ Eq.15

2.6 Spectral Data Measurements

There are two types of measurements taken from the samples. The first measurement is performed over the closed sphere as shown in Figure 3 which we will call total transmittance (T_{total}). In second measurement, sphere is open at direction normal to the sample (see Figure 4), which is used to let the specular portion of incident radiation to escape, this providing diffuse transmittance portion (T_{diff}).

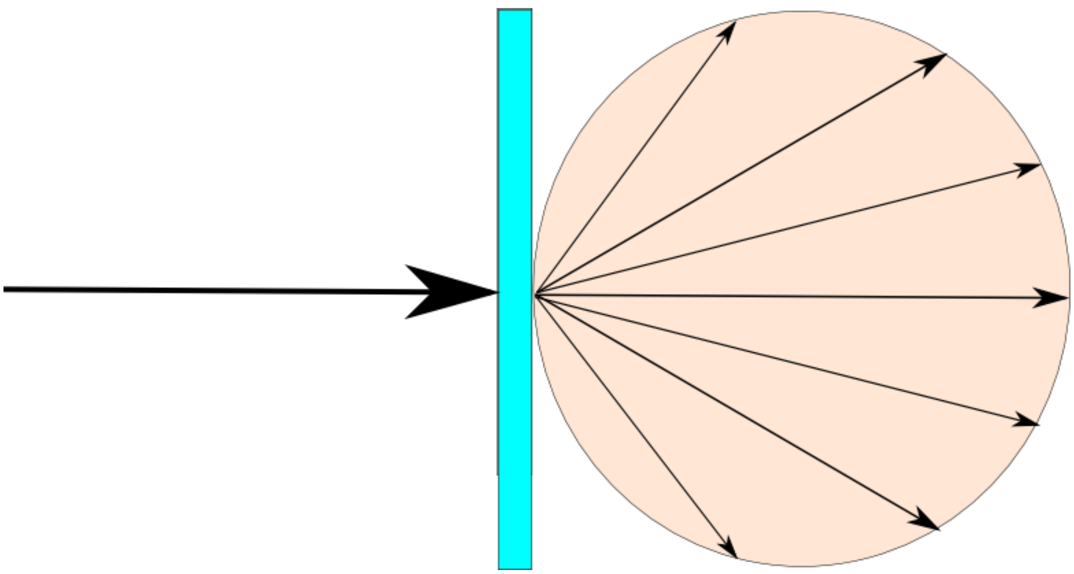

Figure 3: Total Transmittance Measurement

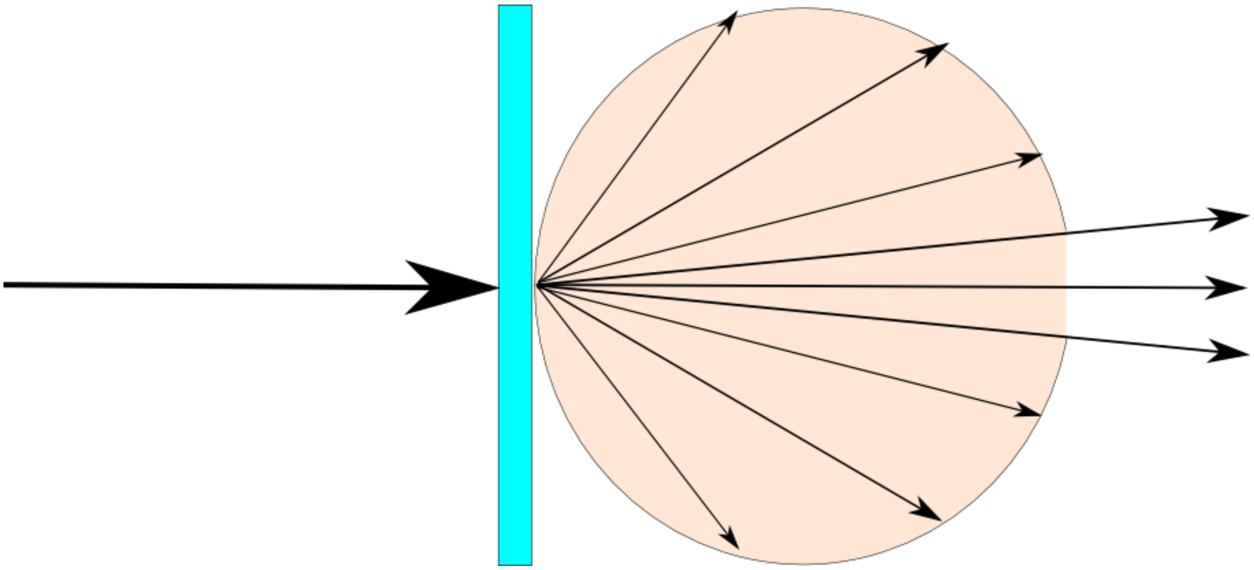

Figure 4: Diffuse Transmittance Measurement

Difference between measured total transmittance and diffuse transmittance is specular transmittance:

$$T_{i,specular} = T_{i,total} - T_{i,diffuse}$$ Eq.16

Note that while Eq.16~~Eq.16~~ is true for each incoming direction, measurement is done for normal incident direction or i=1.

$T_{1,total}$, and $T_{1,diff}$ are measured in spectrophotometer and are reported as a result of measurement for each wavelength. If we consider how these quantities are calculated by the internal instrument's software, we can prove that their difference is equal to the members of diagonal component in a specular matrix (see Section 2.2). The following formulas are applicable to these two quantities:

$$T_{1,total} = \sum_{j=1}^{N} \tau_{j,1} \Lambda_j$$ Eq.17

while for diffuse measurement:

$$T_{1,diff} = \sum_{j=2}^{N} \tau_{j,1} \Lambda_j$$ Eq.18

which leads to the equation for specular transmittance:

$$T_{1,specular} = T_{1,total} - T_{1,diff} = \tau_{1,1} \Lambda_1$$ Eq.19

From which:

$$\tau_{1,1} = \frac{T_{1,total} - T_{1,diff}}{\Lambda_1} = \frac{T_{1,specular}}{\Lambda_1}$$ Eq.20

$$\tau_{i,i} = \frac{T_{i,specular}}{\Lambda_i} \qquad \text{Eq.21}$$

$\tau_{1,1}$ and the rest of $\tau_{i,i}$ are diagonal elements in a specular matrix.

Note that value $T_{1,specular}$ in Eq.20Eq.20 is measured for normal incidence and therefore equation can be applied only to $\tau_{1,1}$. For specular samples, normal incidence measurement is usually the only measurement done. For other angles of incidence, formulas are used that correlate normal to off normal incidence angles (See Reference [1]). It is also possible to measure at different angles of incidence, in which case values do not need to be correlated.

The following set of equations details elements of diffuse matrix, $\tau_{diff,i}$. Rewriting Eq.18Eq.18 for any incidence direction "i":

$$T_{i,diff} + \tau_{diff,i}\Lambda_i = \tau_{diff,i}\sum_{j=1}^{N}\Lambda_j \qquad \text{Eq.22}$$

$$\tau_{diff,i} = \frac{T_{i,diff}}{\pi - \Lambda_i} \qquad \text{Eq.23}$$

where $T_{i,diff}$ is angular dependent.

3. REFERENCES

[1] WINDOW 4.0: Documentation of Calculation Procedures, E.U. Finlayson, D.K. Arasteh, C. Huizenga, M.D. Rubin, and M. S. Reily, LBL-33943, UC-350, July 1993.